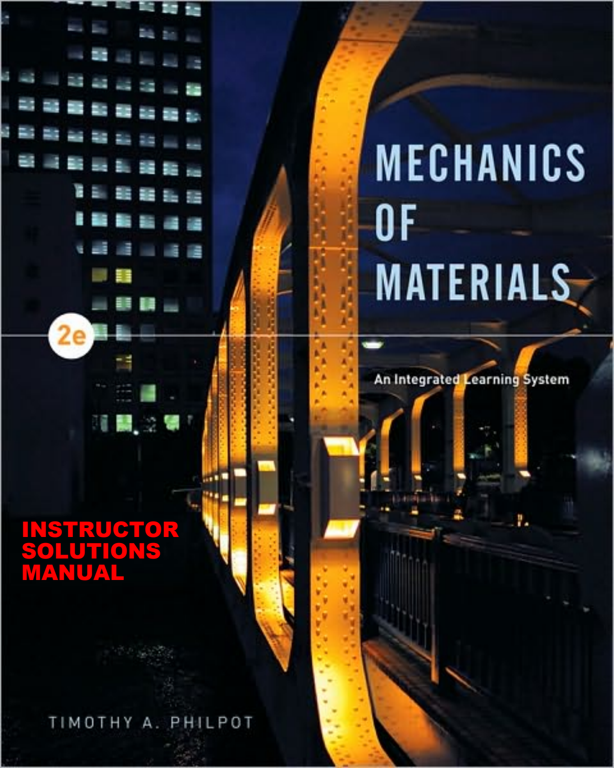

MECHANICS
OF
MATERIALS
2e
An Integrated Learning System
INSTRUCTOR
SOLUTIONS
MANUAL
TIMOTHY A. PHILPOT

1.1 A stainless steel tube with an outside diameter of 60 mm and a wall thickness of 5 mm is used as a compression member. If the normal stress in the member must be limited to 200 MPa, determine the maximum load *P* that the member can support.

Solution

The cross-sectional area of the stainless steel tube is

$$A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}[(60 \text{ mm})^2 - (50 \text{ mm})^2] = 863.938 \text{ mm}^2$$

The normal stress in the tube can be expressed as

$$\sigma = \frac{P}{A}$$

The maximum normal stress in the tube must be limited to 200 MPa. Using 200 MPa as the allowable normal stress, rearrange this expression to solve for the maximum load *P*

$$P_{\text{max}} \leq \sigma_{\text{allow}} A = (200 \text{ N/mm}^2)(863.938 \text{ mm}^2) = 172{,}788 \text{ N} = \boxed{172.8 \text{ kN}}$$ **Ans.**

1.2 A 2024-T4 aluminum tube with an outside diameter of 2.50 in. will be used to support a 27-kip load. If the normal stress in the member must be limited to 18 ksi, determine the wall thickness required for the tube.

Solution

From the definition of normal stress, solve for the minimum area required to support a 27-kip load without exceeding a stress of 18 ksi

$$\sigma = \frac{P}{A} \qquad \therefore A_{\text{min}} \geq \frac{P}{\sigma} = \frac{27 \text{ kips}}{18 \text{ ksi}} = 1.500 \text{ in.}^2$$

The cross-sectional area of the aluminum tube is given by

$$A = \frac{\pi}{4}(D^2 - d^2)$$

Set this expression equal to the minimum area and solve for the maximum inside diameter d

$$\frac{\pi}{4}[(2.50 \text{ in.})^2 - d^2] \geq 1.500 \text{ in.}^2$$

$$(2.50 \text{ in.})^2 - d^2 \geq \frac{4}{\pi}(1.500 \text{ in.}^2)$$

$$(2.50 \text{ in.})^2 - \frac{4}{\pi}(1.500 \text{ in.}^2) \geq d^2$$

$$\therefore d_{\text{max}} \leq 2.08330 \text{ in.}$$

The outside diameter D, the inside diameter d, and the wall thickness t are related by

$$D = d + 2t$$

Therefore, the minimum wall thickness required for the aluminum tube is

$$t_{\text{min}} \geq \frac{D - d}{2} = \frac{2.50 \text{ in.} - 2.08330 \text{ in.}}{2} = 0.20835 \text{ in.} = \boxed{0.208 \text{ in.}}$$ **Ans.**

1.3 Two solid cylindrical rods (1) and (2) are joined together at flange B and loaded, as shown in Fig. P1.3. The diameter of rod (1) is d_1 = 24 mm and the diameter of rod (2) is d_2 = 42 mm. Determine the normal stresses in rods (1) and (2).

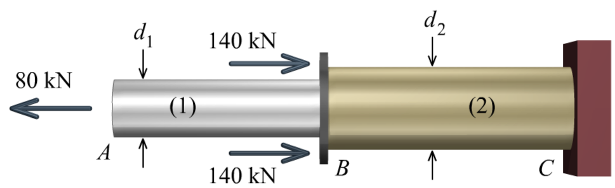
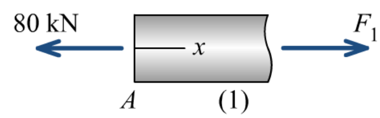


Fig. P1.3

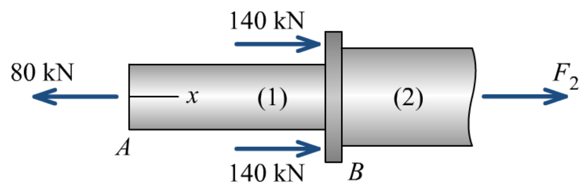
Solution

Cut a FBD through rod (1) that includes the free end of the rod at A. Assume that the internal force in rod (1) is tension. From equilibrium,



$$\Sigma F_x = F_1 - 80 \text{ kN} = 0 \qquad \therefore F_1 = 80 \text{ kN (T)}$$

Next, cut a FBD through rod (2) that includes the free end of the rod A. Assume that the internal force in rod (2) is tension. Equilibrium of this FBD reveals the internal force in rod (2):



$$\Sigma F_x = F_2 + 140 \text{ kN} + 140 \text{ kN} - 80 \text{ kN} = 0 \qquad \therefore F_2 = -200 \text{ kN} = 200 \text{ kN (C)}$$

From the given diameter of rod (1), the cross-sectional area of rod (1) is

$$A_1 = \frac{\pi}{4}(24 \text{ mm})^2 = 452.3893 \text{ mm}^2$$

and thus, the normal stress in rod (1) is

$$\sigma_1 = \frac{F_1}{A_1} = \frac{(80 \text{ kN})(1{,}000 \text{ N/kN})}{452.3893 \text{ mm}^2} = 176.8388 \text{ MPa} = \boxed{176.8 \text{ MPa (T)}}$$ **Ans.**

From the given diameter of rod (2), the cross-sectional area of rod (2) is

$$A_2 = \frac{\pi}{4}(42 \text{ mm})^2 = 1{,}385.4424 \text{ mm}^2$$

Accordingly, the normal stress in rod (2) is

$$\sigma_2 = \frac{F_2}{A_2} = \frac{(-200 \text{ kN})(1{,}000 \text{ N/kN})}{1{,}385.4424 \text{ mm}^2} = -144.3582 \text{ MPa} = \boxed{144.4 \text{ MPa (C)}}$$ **Ans.**

1.4 Two solid cylindrical rods (1) and (2) are joined together at flange *B* and loaded, as shown in Fig. P1.4. If the normal stress in each rod must be limited to 120 MPa, determine the minimum diameter required for each rod.

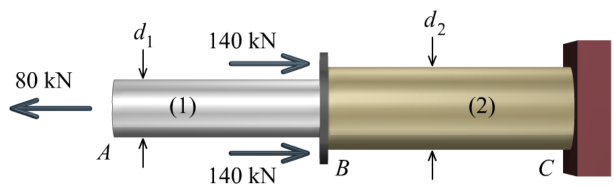
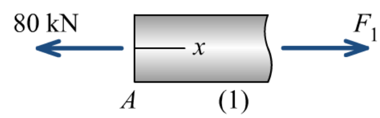


Fig. P1.4

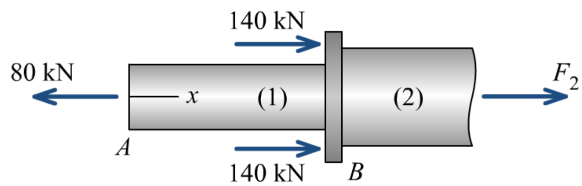
Solution

Cut a FBD through rod (1) that includes the free end of the rod at *A*. Assume that the internal force in rod (1) is tension. From equilibrium,



$$\Sigma F_x = F_1 - 80 \text{ kN} = 0 \qquad \therefore F_1 = 80 \text{ kN (T)}$$

Next, cut a FBD through rod (2) that includes the free end of the rod *A*. Assume that the internal force in rod (2) is tension. Equilibrium of this FBD reveals the internal force in rod (2):



$$\Sigma F_x = F_2 + 140 \text{ kN} + 140 \text{ kN} - 80 \text{ kN} = 0 \qquad \therefore F_2 = -200 \text{ kN} = 200 \text{ kN (C)}$$

If the normal stress in rod (1) must be limited to 120 MPa, then the minimum cross-sectional area that can be used for rod (1) is

$$A_{1,\min} \geq \frac{F_1}{\sigma} = \frac{(80 \text{ kN})(1{,}000 \text{ N/kN})}{120 \text{ N/mm}^2} = 666.6667 \text{ mm}^2$$

The minimum rod diameter is therefore

$$A_{1,\min} = \frac{\pi}{4} d_1^2 \geq 666.6667 \text{ mm}^2 \qquad \therefore d_1 \geq 29.1346 \text{ mm} = \boxed{29.1 \text{ mm}} \qquad \textbf{Ans.}$$

Similarly, the normal stress in rod (2) must be limited to 120 MPa. Notice that rod (2) is in compression. In this situation, we are concerned only with the magnitude of the stress; therefore, we will use the magnitude of F_2 in the calculations for the minimum required cross-sectional area.

$$A_{2,\min} \geq \frac{F_2}{\sigma} = \frac{(200 \text{ kN})(1{,}000 \text{ N/kN})}{120 \text{ N/mm}^2} = 1{,}666.6667 \text{ mm}^2$$

The minimum diameter for rod (2) is therefore

$$A_{2,\min} = \frac{\pi}{4} d_2^2 \geq 1{,}666.6667 \text{ mm}^2 \qquad \therefore d_2 \geq 46.0659 \text{ mm} = \boxed{46.1 \text{ mm}} \qquad \textbf{Ans.}$$

1.5 Two solid cylindrical rods (1) and (2) are joined together at flange B and loaded, as shown in Fig. P1.5. If the normal stress in each rod must be limited to 40 ksi, determine the minimum diameter required for each rod.

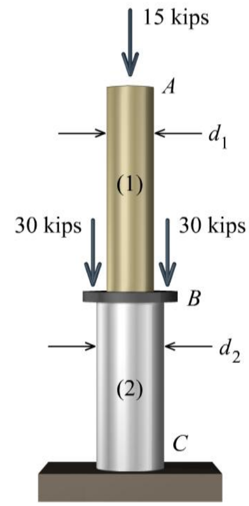
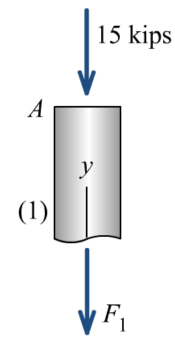


Fig. P1.5

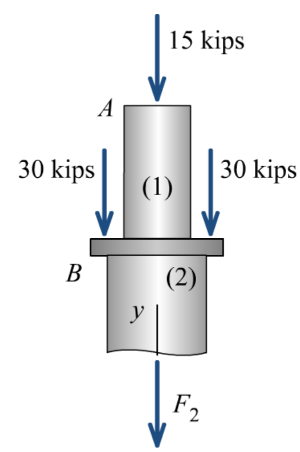
Solution



Cut a FBD through rod (1). The FBD should include the free end of the rod at A. As a matter of course, we will assume that the internal force in rod (1) is tension (even though it obviously will be in compression). From equilibrium,

$$\Sigma F_y = -F_1 - 15 \text{ kips} = 0$$

$$\therefore F_1 = -15 \text{ kips} = 15 \text{ kips (C)}$$



Next, cut a FBD through rod (2) that includes the free end of the rod at A. Again, we will assume that the internal force in rod (2) is tension. Equilibrium of this FBD reveals the internal force in rod (2):

$$\Sigma F_y = -F_2 - 30 \text{ kips} - 30 \text{ kips} - 15 \text{ kips} = 0$$

$$\therefore F_2 = -75 \text{ kips} = 75 \text{ kips (C)}$$

Notice that rods (1) and (2) are in compression. In this situation, we are concerned only with the stress magnitude; therefore, we will use the force magnitudes to determine the minimum required cross-sectional areas. If the normal stress in rod (1) must be limited to 40 ksi, then the minimum cross-sectional area that can be used for rod (1) is

$$A_{1,\min} \ge \frac{F_1}{\sigma} = \frac{15 \text{ kips}}{40 \text{ ksi}} = 0.375 \text{ in.}^2$$

The minimum rod diameter is therefore

$$A_{1,\min} = \frac{\pi}{4} d_1^2 \ge 0.375 \text{ in.}^2 \qquad \therefore d_1 \ge 0.69099 \text{ in.} = \boxed{0.691 \text{ in.}}$$ **Ans.**

Similarly, the normal stress in rod (2) must be limited to 40 ksi, which requires a minimum area of

$$A_{2,\min} \ge \frac{F_2}{\sigma} = \frac{75 \text{ kips}}{40 \text{ ksi}} = 1.875 \text{ in.}^2$$

The minimum diameter for rod (2) is therefore

$$A_{2,\min} = \frac{\pi}{4} d_2^2 \ge 1.875 \text{ in.}^2 \qquad \therefore d_2 \ge 1.545097 \text{ in.} = \boxed{1.545 \text{ in.}}$$ **Ans.**

1.6 Two solid cylindrical rods (1) and (2) are joined together at flange *B* and loaded, as shown in Fig. P1.6. The diameter of rod (1) is 1.75 in. and the diameter of rod (2) is 2.50 in. Determine the normal stresses in rods (1) and (2).

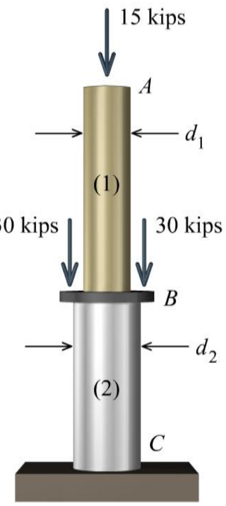
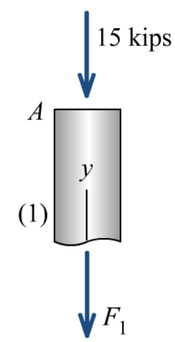


Fig. P1.6

Solution

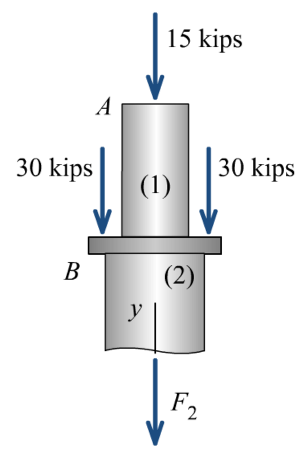
Cut a FBD through rod (1). The FBD should include the free end of the rod at *A*. We will assume that the internal force in rod (1) is tension (even though it obviously will be in compression). From equilibrium,



$$\Sigma F_y = -F_1 - 15 \text{ kips} = 0$$

$$\therefore F_1 = -15 \text{ kips} = 15 \text{ kips (C)}$$

Next, cut a FBD through rod (2) that includes the free end of the rod at *A*. Again, we will assume that the internal force in rod (2) is tension. Equilibrium of this FBD reveals the internal force in rod (2):



$$\Sigma F_y = -F_2 - 30 \text{ kips} - 30 \text{ kips} - 15 \text{ kips} = 0$$

$$\therefore F_2 = -75 \text{ kips} = 75 \text{ kips (C)}$$

From the given diameter of rod (1), the cross-sectional area of rod (1) is

$$A_1 = \frac{\pi}{4}(1.75 \text{ in.})^2 = 2.4053 \text{ in.}^2$$

and thus, the normal stress in rod (1) is

$$\sigma_1 = \frac{F_1}{A_1} = \frac{-15 \text{ kips}}{2.4053 \text{ in.}^2} = -6.23627 \text{ ksi} = \boxed{6.24 \text{ ksi (C)}}$$ **Ans.**

From the given diameter of rod (2), the cross-sectional area of rod (2) is

$$A_2 = \frac{\pi}{4}(2.50 \text{ in.})^2 = 4.9087 \text{ in.}^2$$

Accordingly, the normal stress in rod (2) is

$$\sigma_2 = \frac{F_2}{A_2} = \frac{-75 \text{ kips}}{2.4053 \text{ in.}^2} = -15.2789 \text{ ksi} = \boxed{15.28 \text{ ksi (C)}}$$ **Ans.**

1.7 Axial loads are applied with rigid bearing plates to the solid cylindrical rods shown in Fig. P1.7. The diameter of aluminum rod (1) is 2.00 in., the diameter of brass rod (2) is 1.50 in., and the diameter of steel rod (3) is 3.00 in. Determine the normal stress in each of the three rods.

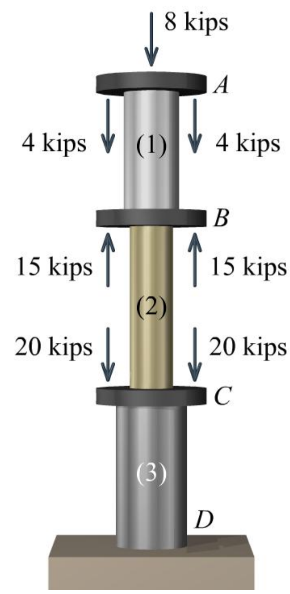
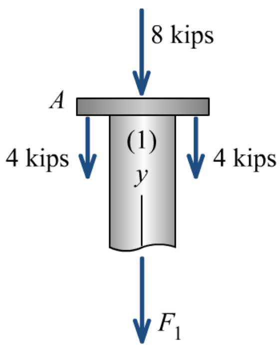


Fig. P1.7

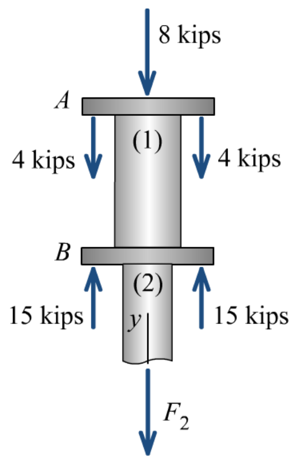
Solution

Cut a FBD through rod (1). The FBD should include the free end *A*. We will assume that the internal force in rod (1) is tension (even though it obviously will be in compression). From equilibrium,

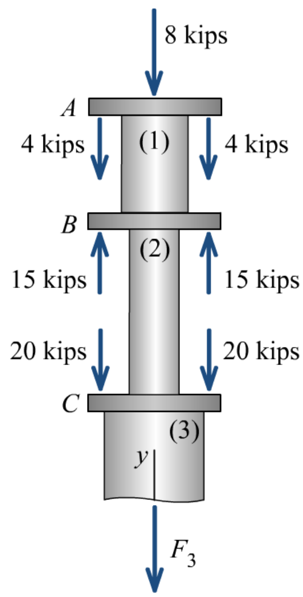
$$\Sigma F_y = -F_1 - 8 \text{ kips} - 4 \text{ kips} - 4 \text{ kips} = 0 \qquad \therefore F_1 = -16 \text{ kips} = 16 \text{ kips (C)}$$



FBD through rod (1)



FBD through rod (2)



FBD through rod (3)

Next, cut a FBD through rod (2) that includes the free end *A*. Again, we will assume that the internal force in rod (2) is tension. Equilibrium of this FBD reveals the internal force in rod (2):

$$\Sigma F_y = -F_2 - 8 \text{ kips} - 4 \text{ kips} - 4 \text{ kips} + 15 \text{ kips} + 15 \text{ kips} = 0 \quad \therefore F_2 = 14 \text{ kips} = 14 \text{ kips (T)}$$

Similarly, cut a FBD through rod (3) that includes the free end *A*. From this FBD, the internal force in rod (3) is:

$$\Sigma F_y = -F_3 - 8 \text{ kips} - 4 \text{ kips} - 4 \text{ kips} + 15 \text{ kips} + 15 \text{ kips} - 20 \text{ kips} - 20 \text{ kips} = 0$$

$$\therefore F_3 = -26 \text{ kips} = 26 \text{ kips (C)}$$

From the given diameter of rod (1), the cross-sectional area of rod (1) is

$$A_1 = \frac{\pi}{4}(2.00 \text{ in.})^2 = 3.1416 \text{ in.}^2$$

and thus, the normal stress in aluminum rod (1) is

$$\sigma_1 = \frac{F_1}{A_1} = \frac{-16 \text{ kips}}{3.1416 \text{ in.}^2} = -5.0930 \text{ ksi} = \boxed{5.09 \text{ ksi (C)}}$$ **Ans.**

From the given diameter of rod (2), the cross-sectional area of rod (2) is

$$A_2 = \frac{\pi}{4}(1.50 \text{ in.})^2 = 1.7671 \text{ in.}^2$$

Accordingly, the normal stress in brass rod (2) is

$$\sigma_2 = \frac{F_2}{A_2} = \frac{14 \text{ kips}}{1.7671 \text{ in.}^2} = 7.9224 \text{ ksi} = \boxed{7.92 \text{ ksi (T)}}$$ **Ans.**

Finally, the cross-sectional area of rod (3) is

$$A_3 = \frac{\pi}{4}(3.00 \text{ in.})^2 = 7.0686 \text{ in.}^2$$

and the normal stress in the steel rod is

$$\sigma_3 = \frac{F_3}{A_3} = \frac{-26 \text{ kips}}{7.0686 \text{ in.}^2} = -3.6782 \text{ ksi} = \boxed{3.68 \text{ ksi (C)}}$$ **Ans.**

1.8 Axial loads are applied with rigid bearing plates to the solid cylindrical rods shown in Fig. P1.8. The normal stress in aluminum rod (1) must be limited to 18 ksi, the normal stress in brass rod (2) must be limited to 25 ksi, and the normal stress in steel rod (3) must be limited to 15 ksi. Determine the minimum diameter required for each of the three rods.

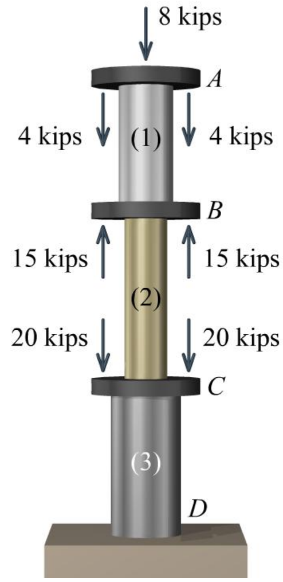
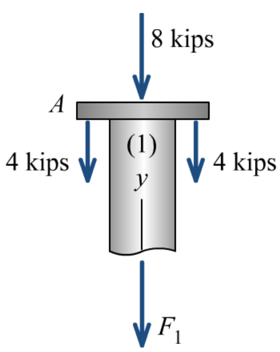


Fig. P1.8

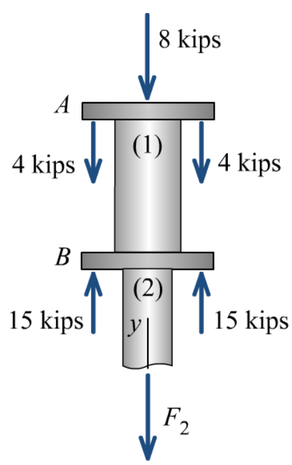
Solution

The internal forces in the three rods must be determined. Begin with a FBD cut through rod (1) that includes the free end *A*. We will assume that the internal force in rod (1) is tension (even though it obviously will be in compression). From equilibrium,

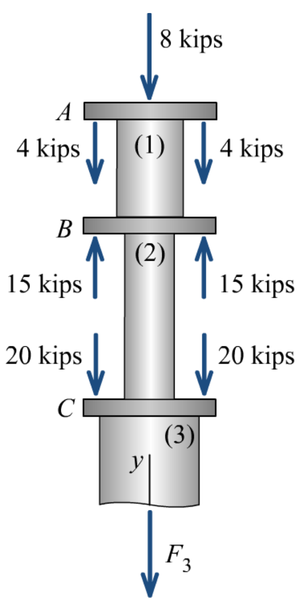
$$\Sigma F_y = -F_1 - 8 \text{ kips} - 4 \text{ kips} - 4 \text{ kips} = 0 \qquad \therefore F_1 = -16 \text{ kips} = 16 \text{ kips (C)}$$



FBD through rod (1)



FBD through rod (2)



FBD through rod (3)

Next, cut a FBD through rod (2) that includes the free end *A*. Again, we will assume that the internal force in rod (2) is tension. Equilibrium of this FBD reveals the internal force in rod (2):

$$\Sigma F_y = -F_2 - 8 \text{ kips} - 4 \text{ kips} - 4 \text{ kips} + 15 \text{ kips} + 15 \text{ kips} = 0 \quad \therefore F_2 = 14 \text{ kips} = 14 \text{ kips (T)}$$

Similarly, cut a FBD through rod (3) that includes the free end *A*. From this FBD, the internal force in rod (3) is:

$$\Sigma F_y = -F_3 - 8 \text{ kips} - 4 \text{ kips} - 4 \text{ kips} + 15 \text{ kips} + 15 \text{ kips} - 20 \text{ kips} - 20 \text{ kips} = 0$$

$$\therefore F_3 = -26 \text{ kips} = 26 \text{ kips (C)}$$

Notice that two of the three rods are in compression. In these situations, we are concerned only with the stress magnitude; therefore, we will use the force magnitudes to determine the minimum required cross-sectional areas, and in turn, the minimum rod diameters. The normal stress in aluminum rod (1) must be limited to 18 ksi; therefore, the minimum cross-sectional area required for rod (1) is

$$A_{1,\min} \geq \frac{F_1}{\sigma_1} = \frac{16 \text{ kips}}{18 \text{ ksi}} = 0.8889 \text{ in.}^2$$

The minimum rod diameter is therefore

$$A_{1,\min} = \frac{\pi}{4} d_1^2 \geq 0.8889 \text{ in.}^2 \qquad \therefore d_1 \geq 1.0638 \text{ in.} = \boxed{1.064 \text{ in.}}$$ **Ans.**

The normal stress in brass rod (2) must be limited to 25 ksi, which requires a minimum area of

$$A_{2,\min} \geq \frac{F_2}{\sigma_2} = \frac{14 \text{ kips}}{25 \text{ ksi}} = 0.5600 \text{ in.}^2$$

which requires a minimum diameter for rod (2) of

$$A_{2,\min} = \frac{\pi}{4} d_2^2 \geq 0.5600 \text{ in.}^2 \qquad \therefore d_2 \geq 0.8444 \text{ in.} = \boxed{0.844 \text{ in.}}$$ **Ans.**

The normal stress in steel rod (3) must be limited to 15 ksi. The minimum cross-sectional area required for this rod is:

$$A_{3,\min} \geq \frac{F_3}{\sigma_3} = \frac{26 \text{ kips}}{15 \text{ ksi}} = 1.7333 \text{ in.}^2$$

which requires a minimum diameter for rod (3) of

$$A_{3,\min} = \frac{\pi}{4} d_3^2 \geq 1.7333 \text{ in.}^2 \qquad \therefore d_3 \geq 1.4856 \text{ in.} = \boxed{1.486 \text{ in.}}$$ **Ans.**

1.9 Two solid cylindrical rods support a load of $P = 50$ kN, as shown in Fig. P1.9. If the normal stress in each rod must be limited to 130 MPa, determine the minimum diameter required for each rod.

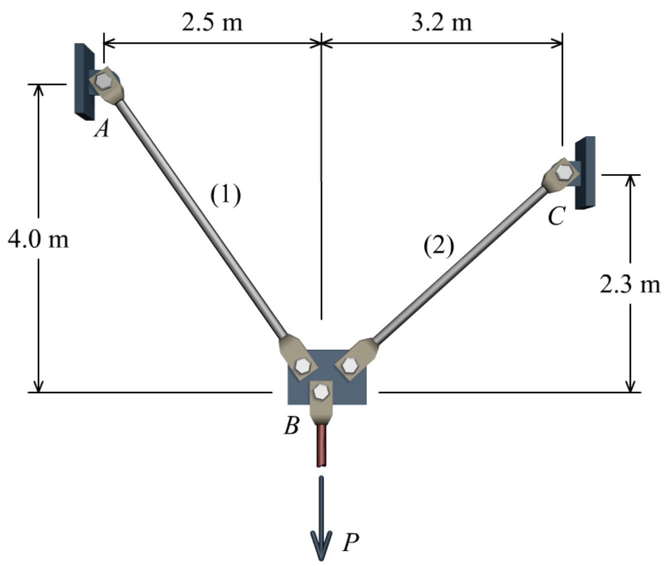


Fig. P1.10

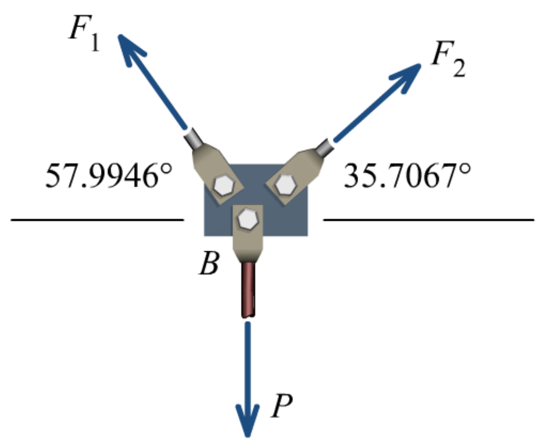
Solution

Consider a FBD of joint B. Determine the angle α between rod (1) and the horizontal axis:

$$\tan\alpha = \frac{4.0\text{ m}}{2.5\text{ m}} = 1.600 \qquad \therefore \alpha = 57.9946°$$

and the angle β between rod (2) and the horizontal axis:

$$\tan\beta = \frac{2.3\text{ m}}{3.2\text{ m}} = 0.7188 \qquad \therefore \beta = 35.7067°$$



Write equilibrium equations for the sum of forces in the horizontal and vertical directions. Note: Rods (1) and (2) are two-force members.

$$\Sigma F_x = F_2\cos(35.7067°) - F_1\cos(57.9946°) = 0 \tag{a}$$

$$\Sigma F_y = F_2\sin(35.7067°) + F_1\sin(57.9946°) - P = 0 \tag{b}$$

Unknown forces F_1 and F_2 can be found from the simultaneous solution of Eqs. (a) and (b). Using the substitution method, Eq. (b) can be solved for F_2 in terms of F_1:

$$F_2 = F_1\frac{\cos(57.9946°)}{\cos(35.7067°)} \tag{c}$$

Substituting Eq. (c) into Eq. (b) gives

$$F_1\frac{\cos(57.9946°)}{\cos(35.6553°)}\sin(35.7067°) + F_1\sin(57.9946°) = P$$

$$F_1\left[\cos(57.9946°)\tan(35.7067°) + \sin(57.9946°)\right] = P$$

$$\therefore F_1 = \frac{P}{\cos(57.9946°)\tan(35.7067°) + \sin(57.9946°)} = \frac{P}{1.2289}$$

For the given load of $P = 50$ kN, the internal force in rod (1) is therefore:

$$F_1 = \frac{50\text{ kN}}{1.2289} = 40.6856\text{ kN}$$

Backsubstituting this result into Eq. (c) gives force F_2:

$$F_2 = F_1 \frac{\cos(57.9946°)}{\cos(35.7067°)} = (40.6856 \text{ kN}) \frac{\cos(57.9946°)}{\cos(35.7067°)} = 26.5553 \text{ kN}$$

The normal stress in rod (1) must be limited to 130 MPa; therefore, the minimum cross-sectional area required for rod (1) is

$$A_{1,\min} \geq \frac{F_1}{\sigma_1} = \frac{(40.6856 \text{ kN})(1{,}000 \text{ N/kN})}{130 \text{ N/mm}^2} = 312.9664 \text{ mm}^2$$

The minimum rod diameter is therefore

$$A_{1,\min} = \frac{\pi}{4} d_1^2 \geq 312.9664 \text{ mm}^2 \qquad \therefore d_1 \geq 19.9620 \text{ mm} = \boxed{19.96 \text{ mm}}$$ **Ans.**

The minimum area required for rod (2) is

$$A_{2,\min} \geq \frac{F_2}{\sigma_2} = \frac{(26.5553 \text{ kN})(1{,}000 \text{ N/kN})}{130 \text{ N/mm}^2} = 204.2718 \text{ mm}^2$$

which requires a minimum diameter for rod (2) of

$$A_{2,\min} = \frac{\pi}{4} d_2^2 \geq 204.2718 \text{ mm}^2 \qquad \therefore d_2 \geq 16.1272 \text{ mm} = \boxed{16.13 \text{ mm}}$$ **Ans.**

1.10 Two solid cylindrical rods support a load of $P = 27$ kN, as shown in Fig. P1.10. Rod (1) has a diameter of 16 mm and the diameter of rod (2) is 12 mm. Determine the normal stress in each rod.

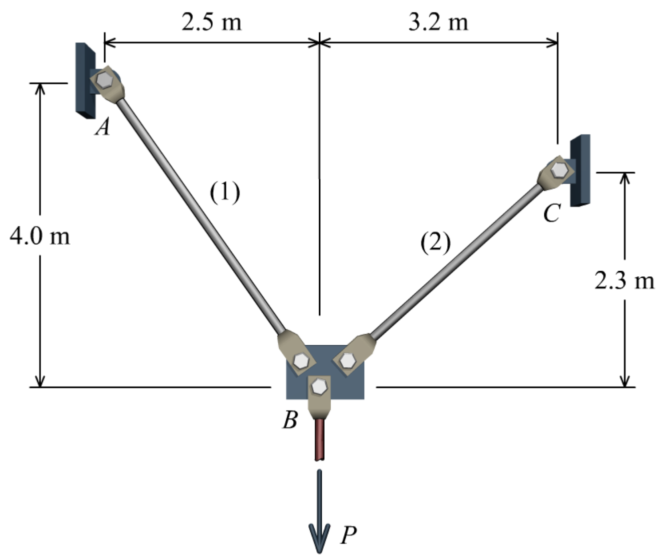


Fig. P1.10

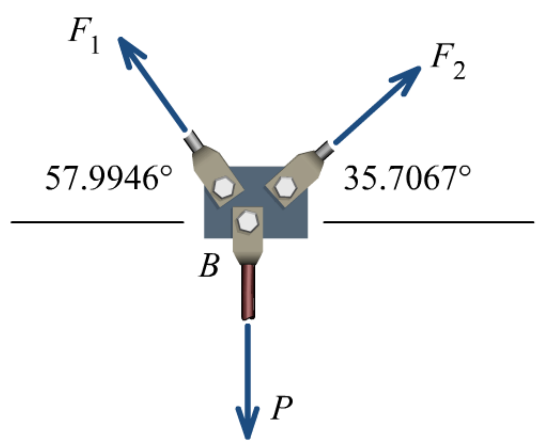
Solution

Consider a FBD of joint *B*. Determine the angle α between rod (1) and the horizontal axis:

$$\tan\alpha = \frac{4.0\text{ m}}{2.5\text{ m}} = 1.600 \qquad \therefore \alpha = 57.9946°$$

and the angle β between rod (2) and the horizontal axis:

$$\tan\beta = \frac{2.3\text{ m}}{3.2\text{ m}} = 0.7188 \qquad \therefore \beta = 35.7067°$$



Write equilibrium equations for the sum of forces in the horizontal and vertical directions. Note: Rods (1) and (2) are two-force members.

$$\Sigma F_x = F_2\cos(35.7067°) - F_1\cos(57.9946°) = 0 \tag{a}$$

$$\Sigma F_y = F_2\sin(35.7067°) + F_1\sin(57.9946°) - P = 0 \tag{b}$$

Unknown forces F_1 and F_2 can be found from the simultaneous solution of Eqs. (a) and (b). Using the substitution method, Eq. (b) can be solved for F_2 in terms of F_1:

$$F_2 = F_1\frac{\cos(57.9946°)}{\cos(35.7067°)} \tag{c}$$

Substituting Eq. (c) into Eq. (b) gives

$$F_1\frac{\cos(57.9946°)}{\cos(35.6553°)}\sin(35.7067°) + F_1\sin(57.9946°) = P$$

$$F_1\left[\cos(57.9946°)\tan(35.7067°) + \sin(57.9946°)\right] = P$$

$$\therefore F_1 = \frac{P}{\cos(57.9946°)\tan(35.7067°) + \sin(57.9946°)} = \frac{P}{1.2289}$$

For the given load of $P = 27$ kN, the internal force in rod (1) is therefore:

$$F_1 = \frac{27\text{ kN}}{1.2289} = 21.9702\text{ kN}$$

Backsubstituting this result into Eq. (c) gives force F_2:

$$F_2 = F_1 \frac{\cos(57.9946°)}{\cos(35.7067°)} = (21.9702 \text{ kN}) \frac{\cos(57.9946°)}{\cos(35.7067°)} = 14.3399 \text{ kN}$$

The diameter of rod (1) is 16 mm; therefore, its cross-sectional area is:

$$A_1 = \frac{\pi}{4}(16 \text{ mm})^2 = 201.0619 \text{ mm}^2$$

and the normal stress in rod (1) is:

$$\sigma_1 = \frac{F_1}{A_1} = \frac{(21.9702 \text{ kN})(1{,}000 \text{ N/kN})}{201.0619 \text{ mm}^2} = 109.2710 \text{ N/mm}^2 = \boxed{109.3 \text{ MPa (T)}}$$ **Ans.**

The diameter of rod (2) is 12 mm; therefore, its cross-sectional area is:

$$A_2 = \frac{\pi}{4}(12 \text{ mm})^2 = 113.0973 \text{ mm}^2$$

and the normal stress in rod (2) is:

$$\sigma_2 = \frac{F_2}{A_2} = \frac{(14.3399 \text{ kN})(1{,}000 \text{ N/kN})}{113.0973 \text{ mm}^2} = 126.7924 \text{ N/mm}^2 = \boxed{126.8 \text{ MPa (T)}}$$ **Ans.**

1.11 A simple pin-connected truss is loaded and supported as shown in Fig. P1.11. All members of the truss are aluminum pipes that have an outside diameter of 4.00 in. and a wall thickness of 0.226 in. Determine the normal stress in each truss member.

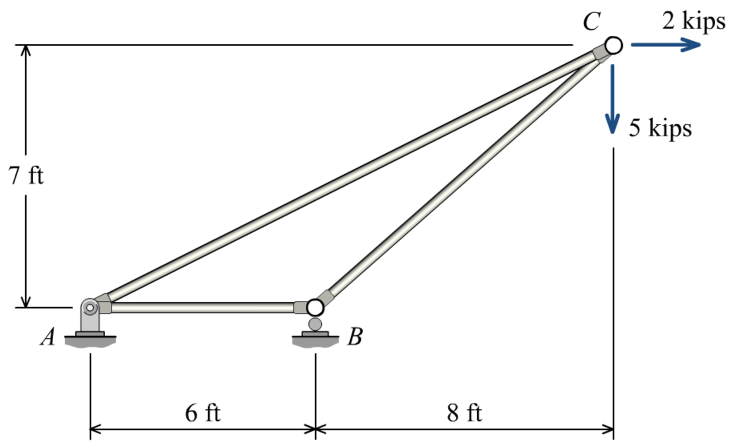
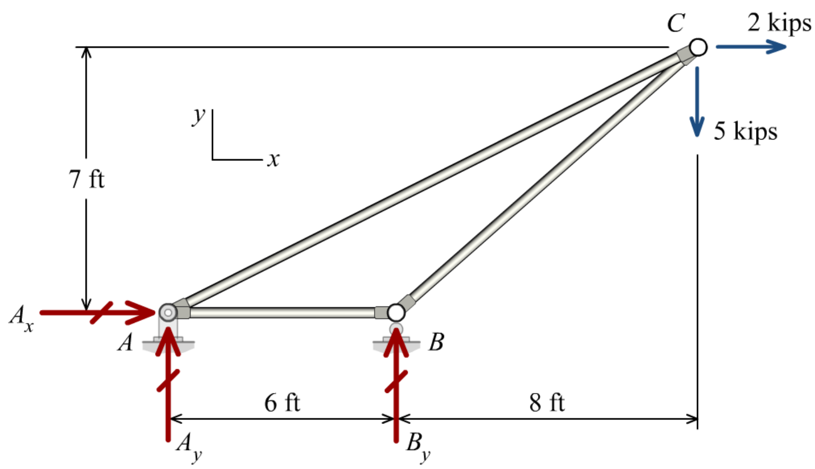


Fig. P1.11

Solution

Overall equilibrium:

Begin the solution by determining the external reaction forces acting on the truss at supports *A* and *B*. Write equilibrium equations that include all *external* forces. Note that only the external forces (i.e., loads and reaction forces) are considered at this time. The internal forces acting in the truss members will be considered after the external reactions have been computed. The free-body diagram (FBD) of the entire truss is shown. The following equilibrium equations can be written for this structure:



$$\Sigma F_x = A_x + 2 \text{ kips} = 0$$

$$\therefore A_x = -2 \text{ kips}$$

$$\Sigma M_A = B_y(6 \text{ ft}) - (5 \text{ kips})(14 \text{ ft}) - (2 \text{ kips})(7 \text{ ft}) = 0$$

$$\therefore B_y = 14 \text{ kips}$$

$$\Sigma F_y = A_y + B_y - 5 \text{ kips} = 0$$

$$\therefore A_y = -9 \text{ kips}$$

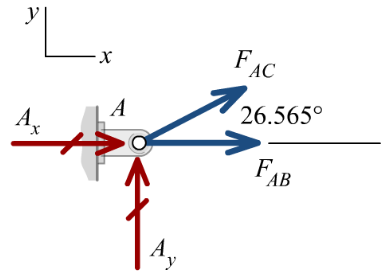
Method of joints:

Before beginning the process of determining the internal forces in the axial members, the geometry of the truss will be used to determine the magnitude of the inclination angles of members *AC* and *BC*. Use the definition of the tangent function to determine θ_{AC} and θ_{BC}:

$$\tan\theta_{AC} = \frac{7 \text{ ft}}{14 \text{ ft}} = 0.50 \qquad \therefore \theta_{AC} = 26.565°$$

$$\tan\theta_{BC} = \frac{7 \text{ ft}}{8 \text{ ft}} = 0.875 \qquad \therefore \theta_{BC} = 41.186°$$

Joint A:
Begin the solution process by considering a FBD of joint *A*. Consider only those forces acting directly on joint *A*. In this instance, two axial members, *AB* and *AC*, are connected at joint *A*. Additionally, two reaction forces, A_x and A_y, act at joint *A*. Tension forces will be assumed in each truss member.

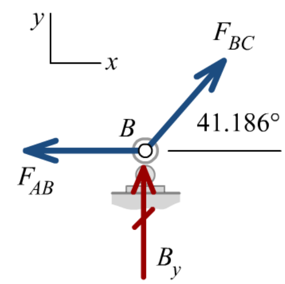


$$\Sigma F_x = F_{AC}\cos(26.565°) + F_{AB} + A_x = 0 \qquad \text{(a)}$$
$$\Sigma F_y = F_{AC}\sin(26.565°) + A_y = 0 \qquad \text{(b)}$$

Solve Eq. (b) for F_{AC}:
$$F_{AC} = -\frac{A_y}{\sin(26.565°)} = -\frac{-9\text{ kips}}{\sin(26.565°)} = 20.125\text{ kips}$$

and then compute F_{AB} using Eq. (a):
$$F_{AB} = -F_{AC}\cos(26.565°) - A_x$$
$$= -(20.125\text{ kips})\cos(26.565°) - (-2\text{ kips}) = -16.000\text{ kips}$$

Joint B:
Next, consider a FBD of joint *B*. In this instance, the equilibrium equations associated with joint *B* seem easier to solve than those that would pertain to joint *C*. As before, tension forces will be assumed in each truss member.



$$\Sigma F_x = -F_{AB} + F_{BC}\cos(41.186°) = 0 \qquad \text{(c)}$$
$$\Sigma F_y = F_{BC}\sin(41.186°) + B_y = 0 \qquad \text{(d)}$$

Solve Eq. (d) for F_{BC}:
$$F_{BC} = -\frac{B_y}{\sin(41.186°)} = -\frac{14\text{ kips}}{\sin(41.186°)} = -21.260\text{ kips}$$

Eq. (c) can be used as a check on our calculations:
$$\Sigma F_x = -F_{AB} + F_{BC}\cos(41.186°)$$
$$= -(-16.000\text{ kips}) + (-21.260\text{ kips})\cos(41.186°) = 0 \qquad \textbf{Checks!}$$

Section properties:
For each of the three truss members:

$$d = 4.00\text{ in.} - 2(0.226\text{ in.}) = 3.548\text{ in.} \qquad A = \frac{\pi}{4}\left[(4.00\text{ in.})^2 - (3.548\text{ in.})^2\right] = 2.67954\text{ in.}^2$$

Normal stress in each truss member:

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{-16.000\text{ kips}}{2.67954\text{ in.}^2} = -5.971\text{ ksi} = \boxed{5.97\text{ ksi (C)}} \qquad \textbf{Ans.}$$

$$\sigma_{AC} = \frac{F_{AC}}{A_{AC}} = \frac{20.125\text{ kips}}{2.67954\text{ in.}^2} = 7.510\text{ ksi} = \boxed{7.51\text{ ksi (T)}} \qquad \textbf{Ans.}$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{-21.260\text{ kips}}{2.67954\text{ in.}^2} = -7.934\text{ ksi} = \boxed{7.93\text{ ksi (C)}} \qquad \textbf{Ans.}$$